22. Convolution Properties

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$$x(t) * y(t) = y(t) * x(t)$$

$$x(t) * [y(t) * z(t)] = [x(t) * y(t)] * z(t)$$

$$x(t) * y(t) = z(\tau)$$

$$x(t - t_0) * y(t - t_1) = z(\tau - t_0 - t_1)$$

Q5. Determine

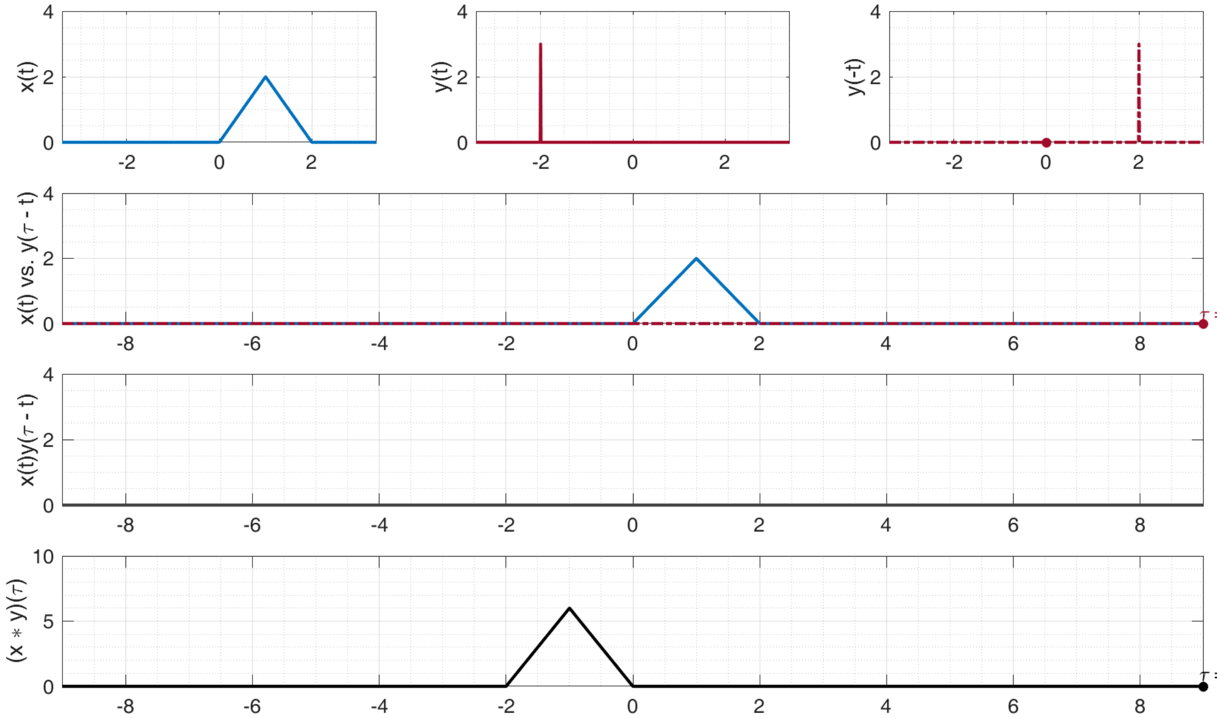
$$x(t) * A\,\delta(t - t_0)$$

Q5. Solution. Substituting in the integral definition of convolution

$$x(t) * A\,\delta(t - t_0) = \int_{-\infty}^{\infty} x(t)\,A\,\delta(\tau - t - t_0)\,dt$$

$$x(t) * A\,\delta(t - t_0) = A \int_{-\infty}^{\infty} x(t)\,\delta\big((\tau - t_0) - t\big)\,dt = A\,x(\tau - t_0)$$

Because $\delta\left(\frac{t-(\tau-t_0)}{-1}\right)$ is an impulse located at $t = (\tau - t_0)$, then according to the sampling property of the impulse, the integral gives the value of $x(t)$ at that location of the impulse, which is $x\big(t = (\tau - t_0)\big) = x(\tau - t_0)$.



$$x(t) * A\,\delta(t - t_0) = A\,x(\tau - t_0)$$

Property #1: The convolution of any signal $x(t)$ with an impulse (Dirac delta signal) $A\,\delta(t - t_0)$ results in $A\,x(\tau - t_0)$. In other words, the resulting signal has the same shape as $x(t)$, except that its amplitude is scaled by the area of the impulse and it is also time shifted from its current location by the shift amount (and direction) of the impulse compared to the origin.

Also, the convolution of two rect( ) signals of the same width gives a triangular Δ( ) signal of double the width, while the convolution of two rect( ) signals of different widths gives a trapezoid signal (Do you know the trapezoid width? How about its location?)

Property #2: Convolution operation is commutative (order of operands does not matter)

$$x(t) * y(t) = y(t) * x(t)$$

Property #3: Convolution can be distributed over addition

$$x(t) * [y(t) + z(t)] = x(t) * y(t) + x(t) * z(t)$$

Property #4: Convolution satisfies the associative property

$$x(t) * [y(t) * z(t)] = [x(t) * y(t)] * z(t)$$

These can be easily proven by going back to the integral definition of convolution.

We can use the commutative property of convolution to our advantage by computing $x(t) * y(t)$ or $y(t) * x(t)$, whichever is simpler. Typically, convolution is easier to perform graphically if we invert (time-reverse) the simpler of the two signals.

Other properties can also be used to simplify calculations.

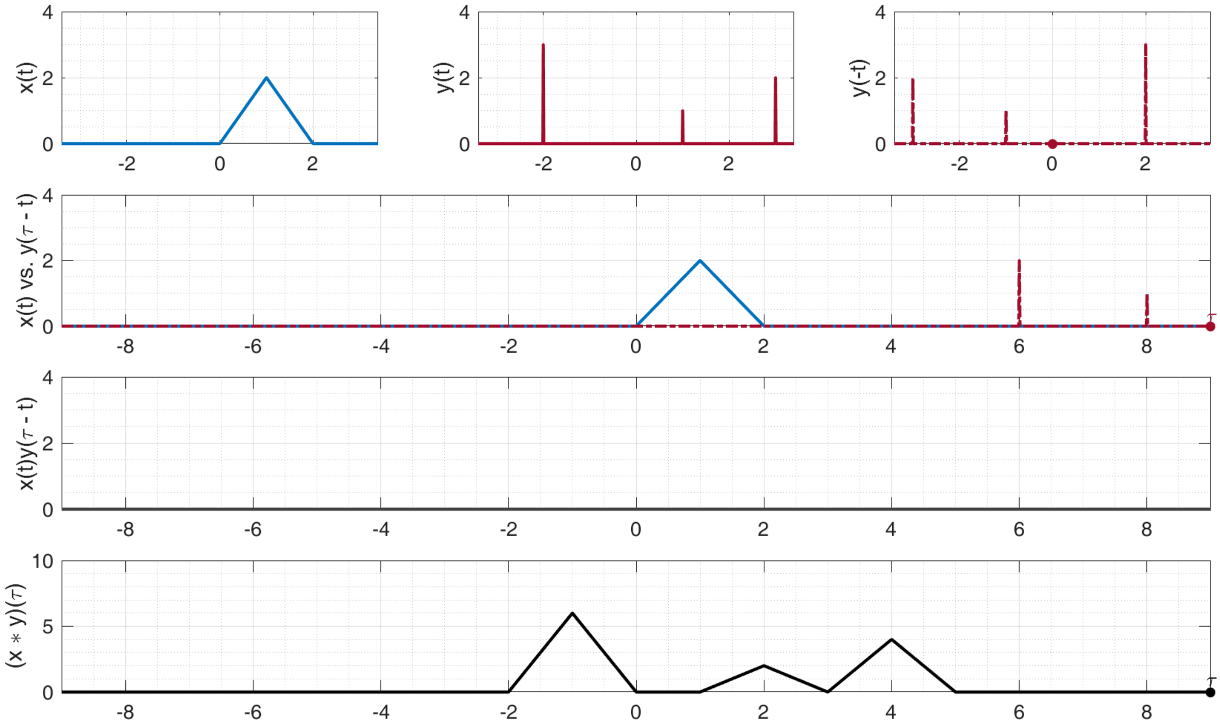
Q6. Determine

$$z(\tau) = 2\,\Delta(t-1) * [3\,\delta(t+2) + \delta(t-1) + 2\,\delta(t-3)]$$

Q6. Solution. Distributing convolution over addition we get

$$z(\tau) = [2\,\Delta(t-1) * 3\,\delta(t+2)] + [2\,\Delta(t-1) * \delta(t-1)] + [2\,\Delta(t-1) * 2\,\delta(t-3)]$$

$$z(\tau) = 6\,\Delta(\tau+1) + 2\,\Delta(\tau-2) + 4\,\Delta(\tau-4)$$



Property #5: Convolution result stays intact with time shifts. In other words, if

$$x(t) * y(t) = z(\tau)$$

Then

$$x(t - t_0) * y(t - t_1) = z(\tau - t_0 - t_1)$$

Also

$$x(t - t_0) * y(t) = x(t) * y(t - t_0) = z(\tau - t_0)$$

If the width of $x(t)$ is T_1 (finite) and the width of $y(t)$ is T_2 (also finite), then the width of convolving the two signal $x(t) * y(t)$ is $T_1 + T_2$.